Deep Learning
Exercise "Automatic Differentiation"

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Exercise 1 - Computation graph

Draw the computation graph for the function

$f(w_1, w_2) = w_2 * sin(p_2 * (p_1 * w_1 + w_2)^2)$

Enumerate the computation nodes in the graph with the names w_1, ...w_8.

Then define a corresponding function f(w1,w2) in Python, where each computation step w_3, ..., w_8 of the computation graph is implemented in Python.

Test your implementation of f with f(3,4). With parameter values $p1 = 1.2345$ and $p2 = 9.8765$ it should return f(3,4)=3.917768.

Exercise 2 - Numerical differentiation

Now define a function numdiff_dfdw1(w1,w2) that computes the derivative of f regarding w1 using numerical differentiation.

Also define a function numdiff_dfdw2(w1,w2) that computes the derivative of f regarding w2 using numerical differentiation.

Check both derivatives at (w1,w2)=(3,4).

In my case, I got this result:

```
Numerical differentiation --> dfdw1(3,4) = -151.64613229501356
Numerical differentiation --> dfdw2(3,4) = -121.85007402054637
```

Exercise 3 - Manual differentiation

Now check how good the numerical differentiation is by computing the derivative of f regarding w1 and w2 manually by making use of differentiation rules.

In my case, I got this result for (w1,w2)=(3,4):

```
Manual differentiation --> dfdw1(3,4) = -151.57699628123763
Manual differentiation --> dfdw2(3,4) = -121.8046780416139
```

Exercise 4 - Reverse-Mode differentiation

Now compute the derivatives $\overline{w}_i = \frac{df}{dw_i} = \frac{dw_8}{dw_i}$ for all computation nodes and write each $\overline{w}_i$ into the picture of your computation graph beside to the corresponding computation node.

Then augment your Python implementation of the function f by adding code that computes these derivatives $\overline{w}_i$.

Use this code to compute the derivatives $\frac{df}{dw_1}(3,4)$ and $\frac{df}{dw_2}(3,4)$ using reverse-mode differentiation.

```
Reverse-Mode Autodiff --> dfdw1(3,4) = ...
Reverse-Mode Autodiff --> dfdw2(3,4) = ...
```

Compare the resulting derivatives with the results from the manual and the numerical differentiation.